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Short communication

Rayleigh–Benard oscillatory natural convection of liquid gallium heated from below

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Abstract

The average rate of heat transfer as well as the oscillatory span-widths of the Nusselt number and the Rayleigh number were measured for natural convection in a layer of liquid gallium ($Pr = 0.023$) heated from below electrically and cooled from above with running water. These quantities were determined by measuring the instantaneous temperature difference between the upper and lower horizontal copper plates. The time-averaged Nusselt number agreed well with the previous data of Rossby for mercury ($Pr = 0.025$), at comparable values of the Rayleigh number. The oscillations are a consequence of reduced viscous damping for a fluid with low Prandtl number.

Keywords: Rayleigh-benard; Natural convection; Liquid gallium

1. Introduction

Rayleigh–Benard natural convection in a horizontal layer of fluid between two solid surfaces was studied extensively after Rayleigh [1], by Jeffreys [2], Pellow and Southwell [3] and others for ordinary fluids and has been studied theoretically by Busse & Clever [4,5], but the experimental measurement for low Prandtl number (Pr) fluids are rather limited. Rossby [6] reported time-averaged Nusselt number (Nu) for mercury ($Pr = 0.025$) for Rayleigh number (Ra) up to 4×10^5.

In the recent development in manufacturing single crystals from a molten fluid, the oscillatory characteristics of liquid metal has been recognized to affect the quality of the solidified products significantly [7].

The present paper reports the results of an experimental investigation of the oscillatory natural convection in a bounded horizontal layer of gallium ($Pr = 0.023$) heated from below.

2. Experimental apparatus and conditions

Fig. 1 shows the overall experimental set-up. The convective layer is $100 \times 100 \times 10$ mm^3. Electric power to the heating wire under the lower copper plate was supplied through a regulated electric DC power supply (Kikusui Elec. Model PAD) with a watt meter for the nichrom wire which had a resistance of 33 Ω. The upper copper plate is a bottom of $180 \times 130 \times 10$ mm^3 cooling-water jacket. The temperature difference between the upper and lower plates was measured with K-type Chromel–Alumel thermocouples glued on surface of the copper plates and connected to a Yokogawa HR 2300 strip-chart recorder.

The segment of the apparatus inside the hatched lines of Fig. 1 was placed in a small room maintained at a uniform temperature approximately equal to the average of the plate temperatures.

Since gallium is corrosive to copper, the plates were initially coated with thin sprayed layer of Teflon. However, the thermal resistance of this layer proved to be both unknown and significant. Hence, it was replaced with gold plating. The details of the convection chamber including both types of coating, are illustrated in Fig. 2.

Gallium is easily and quickly oxidized in air to form a viscous film. To avoid this difficulty the air in the experimental enclosure was replaced with nitrogen before supplying the gallium through a vinyl tube.

The most critical aspect of the experimental measurements was the determination of the heat flux through the gallium. Heat losses were estimated as suggested by Ozoe and Churchill [8] by inverting the apparatus and thereby

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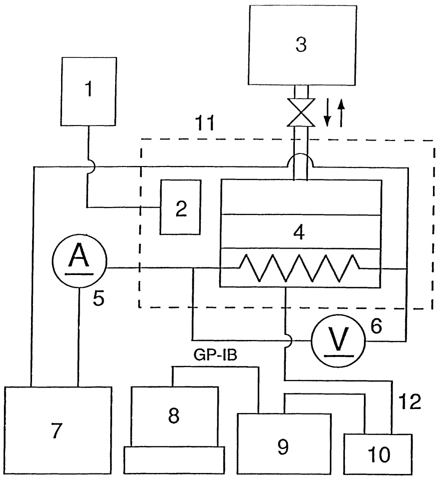

Fig. 1. Schematics of the experimental system: 1 – temperature controller; 2 – temperature-regulated air heated; 3 – constant temperature water bath; 4 – experimental convection layer; 5 – ammeter; 6 – voltmeter; 7 – constant voltage and ampere power supply; 8 – personal computer for data acquisition; 9 – stripchart recorder; 10 – cold junction; 11 – constant-temperature room; 12 – thermocouple.

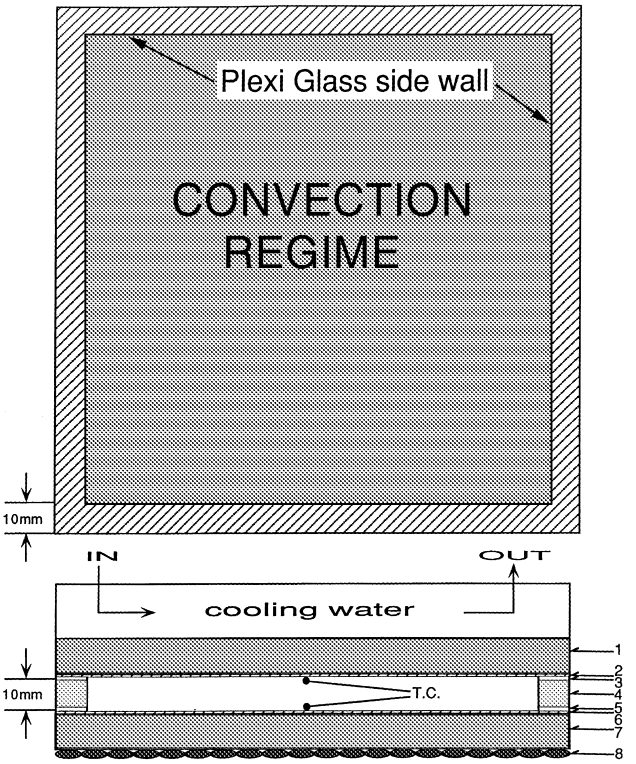

Fig. 2. Details of the convection layer: 1 – cooling copper plate; 2 – teflon-coated layer; 3 – rubber sheet; 4 – plexiglas sidewall; 5 – rubber sheet; 6 – gold-plated layer; 7 – heated copper plate; 8 – nichrome wire heater.

reducing the heat flux through the gallium to that due to pure conduction. This procedure indicated that the heat flux through the gallium was 0.8859 times that supplied to the heater. Thus, the heat losses were 0.1141 times the total input. This result is judged to be sufficiently accurate. More refined measures could be necessary with a less thermally conductive liquid than gallium.

The experimental data were correlated in terms of

$$\mathrm{Ra} = g\beta\ell\,\frac{T_\mathrm{h} - T_\mathrm{c}}{\alpha\nu} \tag{1}$$

and

$$\mathrm{Nu} = \frac{(Q_\mathrm{net}/A)}{\{k(T_\mathrm{h} - T_\mathrm{c})/\ell\}} \tag{2}$$

The relevant physical properties of gallium are listed as follows by Okada and Ozoe [9]:

density	$\rho = 6330 - 0.7717\mathrm{T}$ (Kg m^{-3})
viscosity	$\mu = 0.01207 - 5.754 \times 10^{-5}T + 7.891 \times 10^{-8}T^2$ (Kg m^{-1} s^{-1})
specific heat	$C = 397.6$ (J kg^{-1} K^{-1})
volumetric coefficient of expansion	$\beta = -(\partial\rho/\partial T)/\rho = 0.7717/\rho$ (1/K)
thermal conductivity	$k = -7.448 + 0.1256T$ (W m^{-1} K^{-1})

In evaluating these properties the absolute temperature T was taken to be the average of that of the hot and cold plates.

3. Experimental results

In the preliminary experiments, three thermocouples were imbedded in a heated plate and their outputs of the temperature difference between the cold plate were recorded as shown in Fig. 3. The principal result was that the three thermocouples located at different holes in the heated plate indicated oscillatory temperature fluctuation simultaneously. Afterwards, we changed the location of the thermocouple to the coated copper surface rather than inside the copper plate to measure more accurate temperature difference as shown in Fig. 2.

The experimental results are presented in Table 1 for 20 values of the net heat flux which ranged from 65.7 to 369 W. Values are also given for the maximum and minimum temperature differences due to the oscillations and the corresponding values of Nu and Ra as well as the time-averaged values for these quantities. The time-averaged temperature differences ranged from 2.09 to 9.46°C, those of Ra from 670 to 3040 and those of Nu from 1.003 to 1.19.

In a similar experiment with gallium for a cubic enclosure of $3 \times 3 \times 3$ cm^3, the maximum error in the Nu was 5.7% for the heating rate 12 W for 9 cm^2 as described in detail by Okada and Ozoe [9]. Present data are from 159 to 369 W for 100 square cm. The flux data are similar and the error level should be similar.

Fig. 4 shows the voltage fluctuations of the thermocouple outputs and their frequency for four different net heat inputs. The equivalent scale of the fluctuations in temperature is

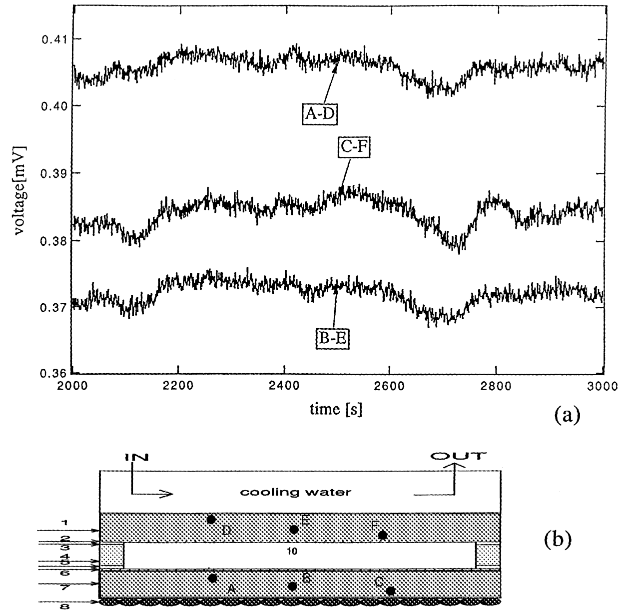

Fig. 3. (a) Transient temperature differences between the heated and cooled plates at three different locations. (b) Locations of three thermocouple holes in both the heated and lower plates. The arrow numbers for the apparatus are the same as those in Fig. 2.

indicated in Fig. 4(c) as an inset. These fluctuations in temperature result from the fluctuations in velocity, which are characteristic of low Pr fluids. The peak frequency of the oscillation increases with the net heat flux. This slow oscillation with a frequency of several minutes would represent the transient change of flow mode which will be clarified by numerical analyses (work in progress).

The ranges of the fluctuations in Nu and Ra, as represented by the X-marks, as well as the time-averaged values, as represented by the solid triangles are shown in Fig. 5. The time-averaged values of Rossby [6] for mercury ($\mathrm{Pr} = 0.025$) are included for comparison. The present values extrapolate more satisfactorily to the critical Rayleigh number of 1708 than do those of Rossby, which suggests that they may be slightly more accurate.

4. Discussion

The present ranges for oscillation in the temperature difference between the heated and the cold plates resulted from the peak and bottom values of the oscillatory data. The temperature oscillation of the low Pr fluid is due to the movement of the liquid metal. At very near the critical state, the plume from a heated plate may start to move slightly, but cooling or heating returns it to the original state. This kind of weak movement would be expected to occur here and there even below the critical state. The critical state may not be at a single value in a mathematical sense but in some range due to the above behavior, as well as the temperature dependence of physical properties for a real fluid especially for low Pr fluid.

Of course, there are a number of reasons for the fluid to move at less than the theoretical critical $\mathrm{Ra_c} = 1708$. For example, the horizontal length is only 100 mm for the depth of 10 mm of fluid layer with plexiglass side walls. This would induce a temperature difference in a horizontal direction in the heated plate. However, the present experiment aims to measure the oscillatory characteristics of a low Pr fluid and not to prove the critical Ra for low Pr.

Table 1
Measured data and dimensionless values

Average T (K)	Q_{net} (W)	$T_\mathrm{h} - T_\mathrm{c}$ (°C)	Nu	Ra	$(T_\mathrm{h} - T_\mathrm{c})_{\mathrm{min}}$	$(T_\mathrm{h} - T_\mathrm{c})_{\mathrm{max}}$	$\mathrm{Ra_{min}}$	$\mathrm{Ra_{max}}$	$\mathrm{Nu_{min}}$	$\mathrm{Nu_{max}}$
308.2	65.7	2.09	1.003	670.0	1.925	2.27	616.0	726.0	0.926	1.091
310.9	142.1	4.56	0.987	1461.0	4.41	4.82	1415.0	1546.0	0.933	1.020
311.3	144.6	4.59	0.994	1474.0	4.45	4.87	1426.0	1562.0	0.938	1.028
311.5	150.7	4.72	1.008	1515.0	4.47	5.02	1433.0	1611.0	0.948	1.065
311.9	159.0	4.98	1.006	1598.0	4.79	5.30	1536.0	1701.0	0.945	1.047
311.7	162.6	5.12	1.004	1643.0	4.73	5.48	1518.0	1757.0	0.939	1.086
312.1	165.5	5.14	1.015	1649.0	4.84	5.45	1552.0	1750.0	0.956	1.078
312.2	170.1	5.29	1.013	1696.0	4.99	5.60	1602.0	1797.0	0.956	1.072
312.7	184.6	5.64	1.029	1810.0	5.26	6.08	1689.0	1951.0	0.954	1.102
313.7	210.0	6.31	1.043	2030.0	5.55	6.81	1783.0	2190.0	0.967	1.186
314.2	228.0	6.67	1.069	2140.0	6.19	7.25	1988.0	2330.0	0.983	1.151
314.7	241.0	7.01	1.071	2250.0	6.32	7.56	2030.0	2430.0	0.993	1.188
315.3	258.0	7.20	1.115	2310.0	6.59	7.93	2120.0	2550.0	1.012	1.218
316.0	276.0	7.60	1.125	2440.0	6.68	8.37	2150.0	2690.0	1.021	1.280
316.7	293.0	8.07	1.123	2600.0	7.01	8.95	2250.0	2880.0	1.013	1.292
316.5	295.0	8.12	1.145	2610.0	7.09	8.90	2280.0	2860.0	1.044	1.311
317.3	306.0	8.29	1.137	2670.0	7.33	9.39	2360.0	3020.0	1.004	1.286
318.2	336.0	8.90	1.163	2860.0	7.70	9.95	2480.0	3200.0	1.040	1.343
318.7	348.0	9.10	1.173	2930.0	7.76	10.14	2500.0	3260.0	1.053	1.377
319.2	369.0	9.46	1.195	3040.0	8.15	10.81	2620.0	3480.0	1.045	1.387

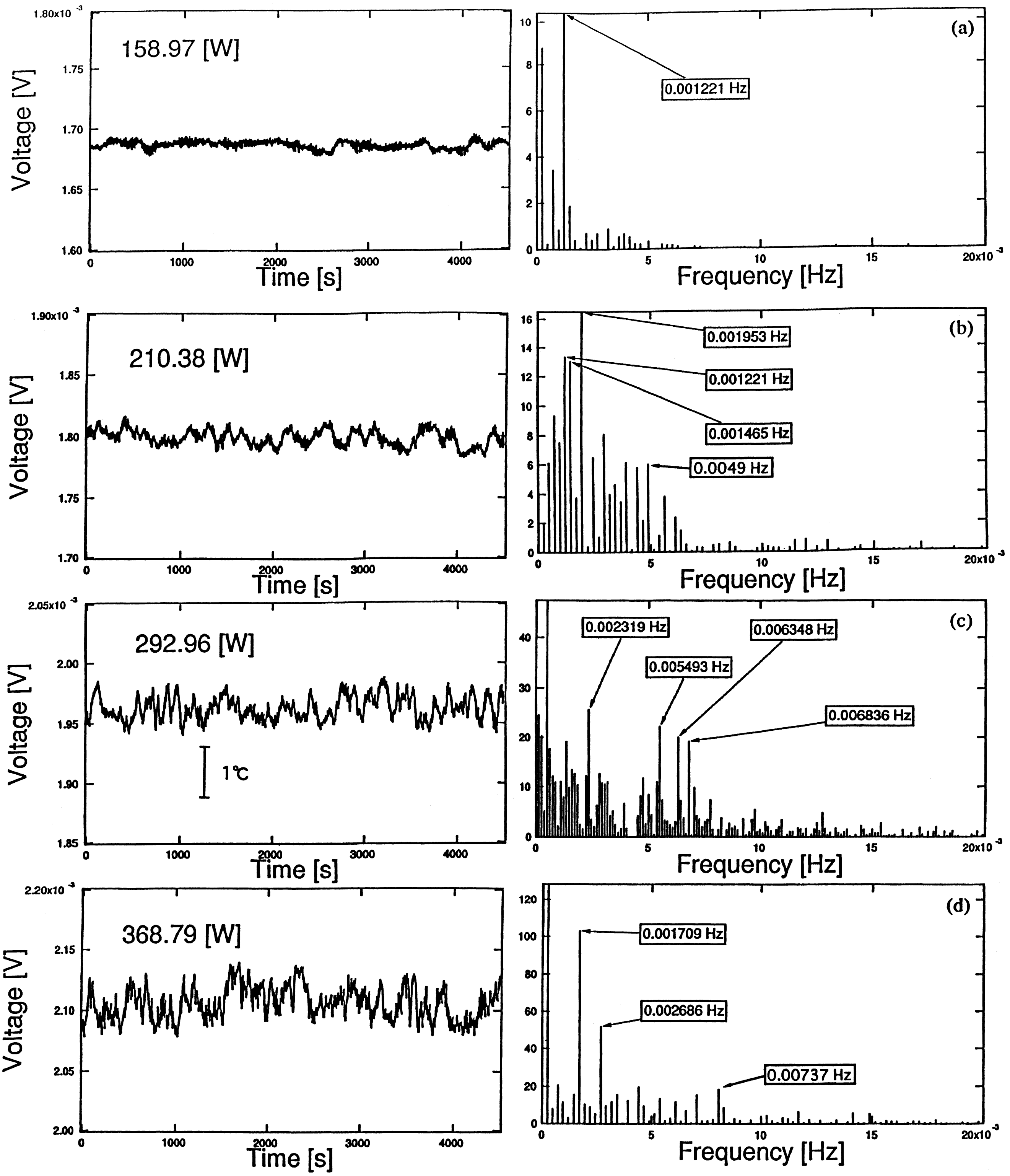

Fig. 4. Transient temperature difference between the heated and cooled walls and their frequency characteristics: (a) $Q_{\mathrm{net}} = 159$ (W); (b) $Q_{\mathrm{net}} = 210$ (W); (c) $Q_{\mathrm{net}} = 293$ (W); (d) $Q_{\mathrm{net}} = 369$ (W).

5. Conclusions

The experimental measurements carried out for a layer of gallium heated from below reveal oscillatory natural convection. The time-averaged values of Nu and Ra agree well with previous measurements of Rossby for mercury. The oscillations are a consequence of reduced viscous damping for a low Pr fluid.

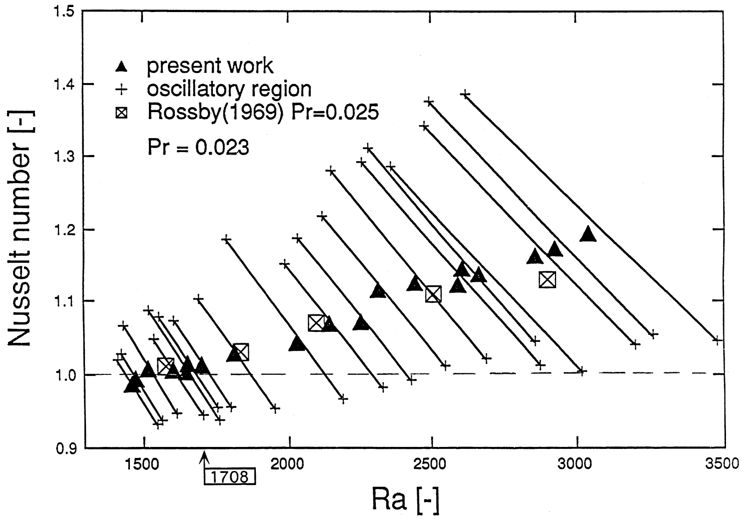

Fig. 5. Summary of the experimental results with the oscillation range in comparison with those by Rossby.

6. Nomenclature

A	heat transfer area (m^{2})
C	specific heat (J kg^{-}1 K^{-1})
g	acceleration due to gravity (m s^{-2})
k	thermal conductivity (J ms^{-1} K^{-1})
ℓ	thick of layer (m)
Nu	average Nusselt number defined by Eq. (2)
Pr	Prandtl number = ν/α
Q_{net}	net heat flux (J s^{-1})
Q_{tot}	total heat flux (J s^{-1})
Ra	Rayleigh number defined by Eq. (1)
T	temperature (K)
T_{c}	cold wall temperature (K)
T_{h}	hot wall temperature (K)

Greek letters

α	thermal diffusivity (m^{2} $^{-1}$s)
β	volumetric coefficient of expansion with temperature (1/K)
ρ	dynamic viscosity (kg m^{-1} s^{-1})
ν	kinematic viscosity = μ/ρ (m^{2} s^{-1})

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